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RAINFALL-RUNOFF RELATIONSHIPS FOR 47 MALAŴI CATCHMENTS

by

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ABSTRACT

Estimates have been made from existing data of annual rainfall, annual runoff and annual evaporation for 47 catchments in Malaŵi. These data have been used to derive estimation models for longterm average runoff and evaporation.

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INTRODUCTION

The work presented in this report arose out of a joint study by the Water Resources Branch of the Department of Lands, Valuation and Water and the Institute of Hydrology, United Kingdom. Basic requirements of this study (Regional analysis of river floods and low flows in Malaŵi, by Drayton et al, 1980) were estimates of the primary hydrological variables - i.e., volumes of rainfall, runoff and actual evaporation. The object of this report is (i) to provide these estimates at 47 stations, and (ii) to derive a relationship between long-term average values of rainfall and runoff.

THE DATA

Only readily available data have been used, and runoff data for 47 stations were available up to 1975 on computer tape. Length of reliable record varies from 5 to 23 years with an average of 12 years, giving a total of 597 station-years. The 47 catchments cover the whole country, but there are some appreciable gaps especially in the Southern Region (see figure 1). Certain runoff stations have recently (1980) been renumbered as part of a rationalisation programme. The catchments affected are principally the Ruo (a major tributary of the Shire River) and the small escarpment catchments between the major drainage basins. Those station numbers affected are listed in Table 1.

Old number	New number
1.D. 3	14.B.3
1.D.10	14.C.3
1.D.11	14.D.2
1.D.14	12.C.4
1.D.23	14.[illegible].2

TABLE 1: Renumbering of flow gauging stations

Rainfall data were available for 227 longterm rainfall stations in the form of monthly and annual totals. Catchment rainfall was estimated from the weighted mean of the annual totals of several raingauges. The weighting was determined mainly by approximate Theissen Polygons but in some cases by subjective consideration of an isohyetal map and/or topography. Raingauges used by their weightings are given in Table 2.

Short gaps of a few months in either rainfall or runoff data were estimated if possible; otherwise the whole year's data were discarded. Where there was some doubt about the data quality, "double-mass curves" (cumulative rainfall-runoff graphs) were plotted to detect errors in the data.

ANALYSIS OF INDIVIDUAL CATCHMENTS

For each catchment, a linear relationship between annual yield or runoff (AY) and annual rainfall (AR) was found by regression.

$$\text{i.e., } AY = a + b\ AR \pm e \text{ mm} \quad (1)$$

$$\text{or} \quad AY = b\ (AR - c) \pm e \text{ mm} \quad (2)$$

where a, b & c are constants
and e is the error term

Figure 2 demonstrates this analysis for station 1D24 (the Kwakwasi river) for 15 years of data.

Bearing in mind that the physical interpretation of regression coefficients should be undertaken with some caution, it is useful to examine the constants a, b and c in more detail. The constant, b, may be interpreted as the proportion of effective rainfall which runs off. The constant, c(-a/b) may be interpreted as the amount of rainfall which is lost (evaporated) before any runoff can occur. This limit condition of zero runoff demonstrates that the linear AY-AR relationship is not physically realistic, because we would always expect some runoff however small the rainfall input. . Some assymptotic relationship which approaches the origin would be closer to reality.

Station No.	No. of gauges	Raingauges & weightings							
1.D.3	4	771-05	(.35)	791-03	(.45)	792-02	(.14)	781-14	(.06)
1.D.10	2	792-07	(.82)	791-06	(.18				
1.D.11	3	793-04	(.33)	791-07	(.33)				
1.D.14	1	792-05	(1.00)						
1.D.23	1	771-05	(1.00)						
1.D.24	2	793-01	(.50)	793-02	(.50)				
1.F.2	2	775-05	(.50)	776-06	(.50)				
1.R.18	2	772-02	(.50)	772-03	(.50)				
2.B.8	1	782-09	(1.00)						
2.B.21	4	781-03	(.26)	781-06	(.46)	782-09	(.21)	781-01	(.07)
2.B.22	2	781-07	(.50)	781-12	(.50)				
2.C.3	3	773-04	(.50)	782-04	(.25)	782-05	(.25)		
2.C.8	2	781-01	(.50)	782-01	(.50)				
3.F.3	2	761-05	(.33)	761-07	(.67)				
4.B.1	8	742-02	(.05)	751-01	(.17)	751-02	(.06)	751-05	(.11)
4.B.1		751-11	(.12)	751-13	(.13)	751-14	(.29)	752-03	(.07)
4.B.3	1	752-03	(1.00)						
4.B.4	2	751-13	(.70)	751-14	(.30)				
4.D.4	2	751-05	(.50)	751-11	(.50)				
4.D.6	1	751-11	(1.00)						
4.E.1	2	751-01	(.50)	751-02	(.50)				
4.E.2	1	751-02	(1.00)						
5.A.8	1	741-09	(1.00)						
5.D.1	3	731-13	(.23)	741-04	(.17)	741-12	(.60)		
5.D.2	2	731-13	(.25)	741-12	(.75)				
5.D.3	1	741-08	(1.00)						

Station No.	No. of gauges	Raingauges & weightings							
5.E.1	1	742-04	(1.00)						
5.E.2	1	741-03	(1.00)						
6.C.1	1	731-13	(1.00)						
6.C.3	1	731-13	(1.00)						
6.F.1	3	731-06	(.10)	722-09	(.10)	Chikwina	(.80)		
6.F.2	3	731-02	(.53)	731-07	(.26)	722-09	(.21)		
6.F.5	3	722-05	(.50)	731-02	(.20)	731-03	(.30)		
6.F.6	2	722-09	(.20)	Chikwina	(.80)				
7.A.3	2	722-13	(.80)	731-11	(.20)				
7.D.3	2	721-13	(.33)	722-09	(.67)				
7.E.2	3	721-03	(.26)	721-13	(.15)	722-06	(.59)		
7.F.1	2	721-01	(.45)	721-11	(.55)			(1955-64)	
7.F.1	3	721-01	(.40)	721-11	(.30)	721-04	(.30)	(1965-74)	
7.F.2	3	721-01	(.44)	721-02	(.33)	721-12	(.23)		
7.G.2	3	721-01	(.50)	721-02	(.20)	721-07	(.30)		
7.G.3	1	721-02	(1.00)						
7.G.11	3	721-02	(.34)	721-07	(.19)	721-08	(.47)		
7.G.14	5	721-03	(.15)	721-01	(.10)	721-12	(.19)	721-13	(.23)
		722-06	(.33)						
7.G.25	3	721-01	(.35)	721-07	(.55)	711-13	(.10)		
8.A.2	3	721-10	(.56)	711-04	(.09)	701-11	(.35)		
8.C.5	2	711-12	(.50)	721-01	(.50)				
8.C.6	2	711-12	(.75)	721-01	(.25)				
9.A.2	3	701-07	(.33)	701-09	(.33)	701-11	(.33)		

TABLE 2: Raingauges and weightings

However, the range of the data for a given catchment, in addition to its quantity and quality, do not permit us to undertake the development of such nonlinear model.

Based on (a) length of record (b) the quality of the basic data and (c) the quality of the rainfall runoff relationship, an overall assessment of each catchments data was made. The 47 stations were categorised as follows:-

Class A :	"Good"	17 stations
Class B :	"Fair"	21 stations
Class C :	"Unacceptable"	9 stations

Table 3 shows the separation into these 3 classes.

Details of each catchment's regression of runoff on rainfall are given in Tables 4A,4B and 4C. Runoff rates (see constant b) for Class A & B stations range from below 10% in the Central Plateau to nearly 90% around the high plateaux. The regressions for all class A and B stations are summarised in figure 3. In this graph, each catchment is represented (a) by its average value of annual rainfall & runoff, and (b) by a straight line of slope b. This figure demonstrates that, in general, catchments with low rainfall have lower rates of runoff (constant b) - this feature supports the nonlinear hypothesis which was suggested earlier but could not be demonstrated in analysis of individual catchments.

Estimates of actual evaporation were made for each year of data from

$$AE = AR - AY \text{ mm} \quad \dots \quad (3)$$

which using the regression equation (1) becomes:-

$$AE = (1 - b)\, AR - a \text{ mm} \quad \dots \quad (4)$$

Due to (i) the strong correlation between runoff & rainfall, & (ii) the constant b not being close to unity, the annual actual evaporation is not expected to be constant for a given catchment. Thus the physical process appears to be one in which the catchment evapotranspiration varies according to the moisture available rather than

STATION NO.	NO. YEARS	DATA QUALITY		
		FLOW	RAIN	OVERALL
1.D.3	16	B	A	B
1.D.10	16	A	C	C
1.D.11	22	A	A	A
1.D.14	20	B	B	B
1.D.23	8	A	A	B
1.D.24	16	A	A	A
1.F.2	15	A	A	B
1.E.18	5	A	A	B
2.B.8	5	A	B	B
2.B.21	13	B	A	A
2.B.22	6	B	A	B
2.C.3	19	A	B	A
2.C.8	14	A	A	A
3.F.3	12	A	A	B
4.B.1	18	A	A	A
4.B.3	17	A	A	B
4.B.4	14	B	B	C
4.D.4	23	A	A	A
4.D.6	10	A	A	B
4,E.1	12	A	A	A
4.E.2	11	B	A	C
5.A.8	13	B	A	B
5.D.1	12	A	A	A
5.D.2	20	A	A	A
5.D.3	11	A	A	B
5.E.1	10	B	A	B
5.E.2	9	A	A	B
6.C.1	14	A	A	A
6.C.3	6	B	A	C
6.F.1	5	A	B	B
6.F.2	12	A	A	A
6.F.5	9	A	A	A
6.F.6	6	A	A	B
7.A.3	15	B	B	C
7.D.3	14	A	A	A

TABLE 3 (PAGE 1 OF 2)

STATION NO.	NO. YEARS	DATA QUALITY		
		FLOW	RAIN	OVERALL
7.E.2	10	A	B	B
7.F.1	19	B	A	B
7.F.2	12	B	A	B
7.G.2	15	A	A	A
7.G.3	10	C	B	C
7.G.11	17	A	A	A
7.G.14	11	A	A	B
7.G.25	15	B	C	C
8.A.2	10	A	A	A
8.C.5	6	B	C	C
8.C.6	6	A	B	B
9.A.2	7	B	A	C

Class A : "Good"

Class B : "Fair"

Class C : "Unacceptable"

TABLE 3 : Data quality assessment

Station number	No. of years	Regression equation	Correlation coefficient
1.D.11	22	AY = .55(AR - 552)	.86
1.D.24	16	AY = .52(AR - 603)	.82
2.B.21	13	AY = .60(AR - 595)	.66
2.C.3	19	AY = .83(AR - 664)	.83
2.C.8	14	AY = .74(AR - 855)	.87
4.B.1	18	AY = .41(AR - 548)	.86
4.D.4	23	AY = .41(AR - 550)	.82
4.E.1	12	AY = .18(AR - 359)	.66
5.D.1	12	AY = .27(AR - 584)	.89
5.D.2	20	AY = .24(AR - 598)	.77
6.C.1	14	AY = .17(AR - 525)	.88
6.F.2	12	AY = .62(AR - 680)	.82
6.F.5	9	AY = .32(AR - 287)	.84
7.D.3	14	AY = .33(AR - 508)	.86
7.G.2	15	AY = .89(AR - 577)	.87
7.G.11	17	AY = .75(AR - 492)	.85
8.A.2	10	AY = .33(AR - 227)	.79

TABLE 4A: Linear regression of AY on AR
for class A stations

Station number	No. of years	Regression equation	Correlation coefficient
1.D.3	16	AY = .44(AR - 672)	.86
1.D.14	20	AY = .33(AR + 890)	.69
1.D.23	8	AY = .66(AR - [illegible]58)	.73
1.F.2	15	AY = .40(AR - 370)	.64
1.E.18	5	AY = .26(AR - 500)	.72
2.B.8	5	AY = .84(AR - 890)	.93
2.E.22	6	AY = .58(AR - 499)	.89
3.F.3	12	AY = .63(AR - 464)	.59
4.B.3	17	AY = .42(AR - 452)	.61
4.D.6	10	AY = .41(AR - 490)	.74
5.A.8	13	AY = .63(AR - 395)	.56
5.D.3	11	AY = .11(AR - 241)	.58
5.E.1	10	AY = .62(AR - 569)	.92
5.E.2	9	AY = .53(AR - 687)	.91
6.F.1	5	AY = .48(AR - 460)	.95
6.F.6	6	AY = .42(AR - 432)	.80
7.E.2	10	AY = .06(AR - 543)	.79
7.F.1	19	AY = .32(AR - 193)	.66
7.F.2	12	AY = .38(AR - 332)	.71
7.G.14	11	AY = .16(AR - 338)	.70
8.C.6	6	AY = .65(AR - 402)	.75

TABLE 4B: Linear regression of AY on AR for class B stations

Station number	No. of years	Regression equation	Correlation coefficient
1.D.10	16	No fit possible	-
4.B.4	14	AY = .46(AR - 400)	.66
4.E.2	11	AY = .17(AR - 328)	.62
6.C.3	6	AY = .29(AR - 345)	.99
7.A.3	15	AY = .13(AR - 408)	.64
7.G.3	10	No fit possible	-
7.C.23	6	AY = 1.10(AR - 600)	.70
8.C.5	6	No fit possible	-
9.A.2	7	AY = .28(AR + 150)	.37

TABLE 4C: Linear regression of AY on AR for class C stations.

Station number	AAR (mm)	AAY (mm)	AAE (mm)	DAMBO
1.D.11	1280	395	885	0
1.D.24	1240	332	908	0
2.B.21	1430	499	931	0
2.C.3	1730	882	848	0
2.C.8	1280	312	968	0
4.B.1	880	133	747	.08
4.D.4	930	155	775	.04
4.E.1	810	82	728	.17
5.D.1	900	83	817	.21
5.D.2	900	73	827	.27
6.C.1	740	37	703	.12
6.F.2	1480	500	980	.06
6.F.5	1090	260	830	.02
7.D.3	1210	232	978	.02
7.G.2	1320	661	659	0
7.G.11	1370	625	745	0
8.A.2	910	227	683	0

TABLE 5A: Estimated primary variables for class A stations.

being constant for a given catchment. Since transpiration is a high proportion of total evapotranspiration, it appears that the vegetation reduces its moisture loss as the moisture available becomes less. However as the average catchment rainfall increases, the slope (b) of the relationship begins to approach unity - so that, in a catchment with very high rainfall, one might expect the actual evaporation not to be very variable (especially in comparison to a catchment with low rainfall).

A REGIONAL MODEL OF ANNUAL RAINFALL AND RUNOFF

The data derived from the analysis of individual catchments were used to develop a regional model relating the long-term averages of annual runoff or yield (AAY) and of rainfall (AAR). This enabled a long-term average of annual (actual) evaporation, AAE, to be estimated from

$$AAE = AAR - AAY \text{ mm} \quad \ldots \quad (5)$$

The average catchment variables are summarised in Tables 5A, 5B, 5C, and plotted on Figure 4. For the purposes of this analysis, the means of the AY's and AR's are assumed to be equal to the longterm means (AAY and AAR). Although some catchments have a relatively short record, this assumption adds no extra error to the analysis.

Figure 4 distinguishes between data quality classes and shows that most "unacceptable" stations are outliers from the main trend. Based on this evidence, class C catchments have been omitted from the remaining analysis.

As a first attempt to examine this trend in long-term averages, a linear relationship was assumed:

$$\text{i.e., } AAY = A + B \times AAR \pm E \text{ mm} \quad \ldots \quad (6)$$
$$\text{or } \quad AAY = B\,(AAR - C) \pm E \text{ mm} \quad \ldots \quad (7)$$

where A, B & C are constants, analogous to a, b & c in equations (1) and (2), and E is the error term.

Station number	AAR (mm)	AAY (mm)	AAE (mm)	DAMBO
1.D.3	1110	193	917	0
1.D.14	2090	978	1112	0
1.D.23	1070	270	800	0
1.F.2	1300	370	930	0
1.K.18	1080	199	881	0
2.B.8	2060	983	1077	0
2.B.22	880	208	672	0
3.F.3	1000	337	663	0
4.B.3	910	192	718	.07
4.D.6	940	182	758	.02
5.A.8	870	298	572	0
5.D.3	710	53	657	.10
5.E.1	930	228	702	.03
5.E.2	910	119	791	.13
6.F.1	1300	403	897	.03
6.F.6	1240	340	900	0
7.E.2	990	40	950	.10
7.F.1	920	233	687	.01
7.F.2	1260	348	912	.01
7.G.14	880	81	799	.06
8.C.6	1060	428	632	.02

TABLE 5B: Estimated primary variables for class B stations

Station number	AAR (mm)	AAY (mm)	AAE (mm)	DAMBO
1.D.10	2760	2840	?	0
4.B.4	830	196	634	.24
4.E.2	940	106	834	.23
6.C.3	673	141	532	.12
7.A.3	790	49	721	.07
7.G.3	1800	385	1415	0
7.G.25	1380	858	522	0
8.C.5	1055	660	350	0
9.A.2	1183	375	808	.01

TABLE 5C: Estimated primary variables for class C stations.

[illegible] (from 0 towards

1) with increasing AAR. This hypothesis is supported by analyses performed on data from South Africa (Midgley and Pitman, 1969), and examination of individual stations in figure 3 reinforces this view. In practice, the presence of dambo in some rivers tends to mask this non-linear effect, turning the data into a form which suggests linearity (see figure 4). A Dambo term is introduced to account for this - DAMBO is defined as the proportion of the area which is covered by Dambo (measured on a 1:50 000 map). Acceptance of the linear relationship relies on an erroneous assumption that <u>all</u> low rainfall areas are affected by Dambo (whereas, in truth, Dambo occurs only in areas of low rainfall, which is a very different and valid assumption). An alternative model, therefore, is given by:

$$AAY = -92 + .16\,AAR + [illegible]8\,AAR^2 - 640\,DAMBO \quad \ldots(10)$$

points indicate the change in position implied by equation 10 if there were no Dambo. The coefficient of the Dambo term is interesting, in that it suggests that there is an extra 640 mm of evaporation from the rest of the catchment - this interpretation is physically sensible. It is recommended that the model given by equation 10 should be used for estimation of AAY for catchments with low rainfall. High rainfall areas will be equally well served by equation 8 or equation 10.

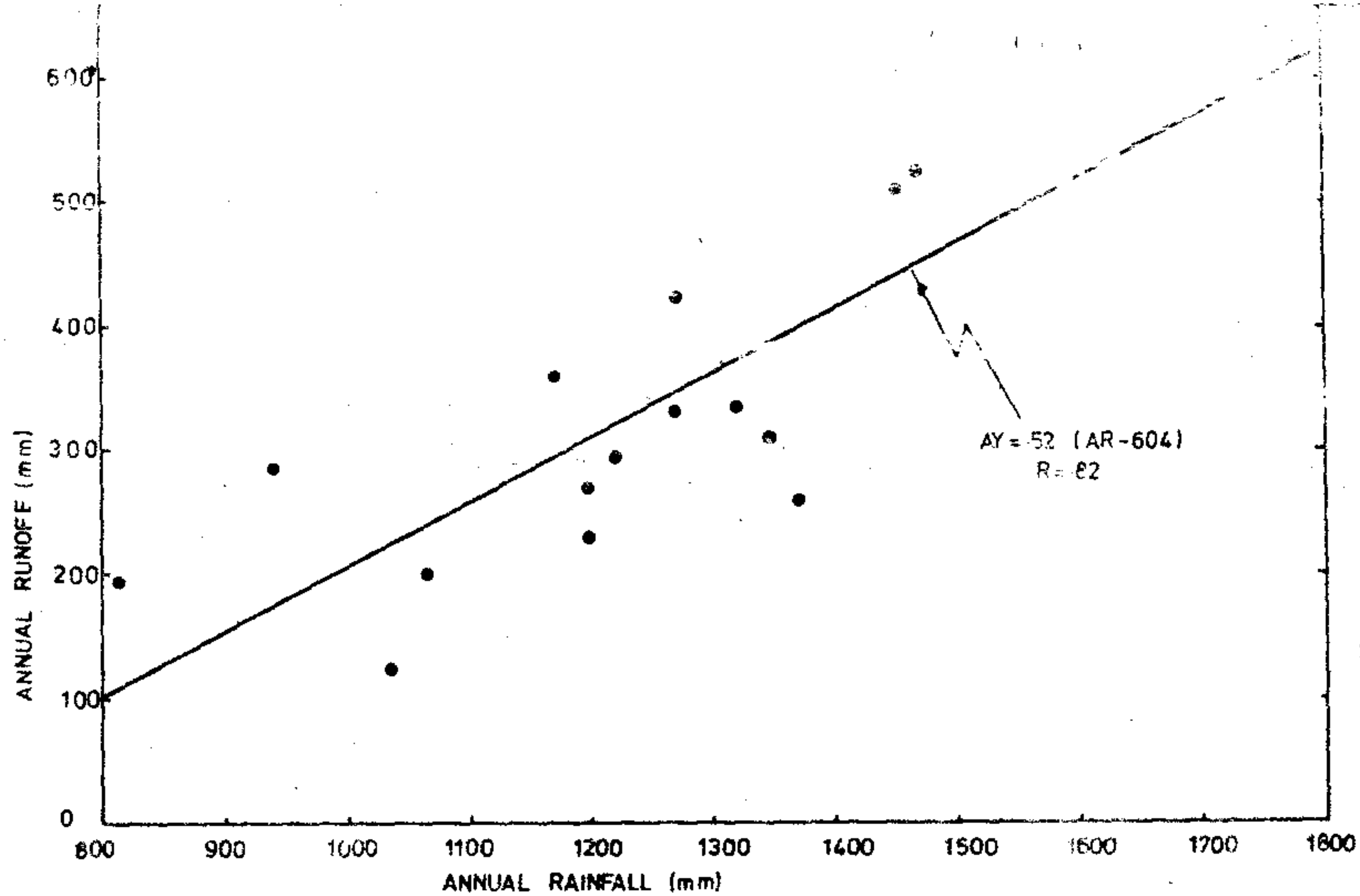

FIGURE 2: Annucl runoff v. cnnucl rcinfcll for 1D24